SELF TEST SEVEN: INTERVAL ESTIMATION

1. The statistic \( \bar{x} \) is a ________ estimate of ________; the statistic \( s \) is a ________ estimate of ________; the statistic \( s/\sqrt{n} \) is a _____ estimate of ________.

2. True or false? Statisticians generally prefer to give point estimates of population values.

3. Explain your answer to q. 2.

4. If we say, for example, “A 95% confidence interval for \( \mu \) is the interval from 40 to 60,” the meaning of the 95% is __________.

5. As confidence increases, everything else remaining the same, the width of a confidence interval __________.

6. For a given confidence level, we can get a more precise estimate only by __________.

7. Find the appropriate z value to use in constructing each of the following confidence intervals: 98%, 95%, 67%.

8. It is known that the standard deviation of IQ in human populations is 15 and that IQ scores are normally distributed. We have a sample of 23 students from Ramapithicus University; in this sample the mean IQ \( \bar{x} = 91 \). A 95% confidence interval for the mean IQ of all RU students is 91 ± _______ or the interval _______ to _______.

9. The standard deviation of diameter for pumpkins is 4", and these diameters are normally distributed. We would like to estimate the average diameter of the pumpkins in a field with a 98% confidence interval, and we require that the error in the estimate be no more than ± 1". To obtain this precision, we will need a sample of \( n = \) _______.

10. We have a sample of 10 watermelons with weights (in lb.) as follows:
    
    \[
    \begin{array}{cccccccccc}
    12 & 34 & 20 & 25 & 18 & 14 & 9 & 12 & 16 & 8 \\
    \end{array}
    \]
    
    Assume that the population is normally distributed. In this problem \( \bar{x} = _____ \), \( s = _____ \) and \( s/\sqrt{n} = _____ \). You should use a _____ value in calculating a confidence interval. A 90% confidence interval for the mean weight of the population of watermelons will be the interval from _______ to _______.

11. We surveyed a sample of 431 ASU students and found that in our sample, 126 preferred summer school to a regular semester. In our sample, \( p = \) _______. An estimate of the standard deviation of the sampling distribution of \( p \), \( s_p = \) _______. The 95% confidence interval for the proportion preferring summer school is _______ ± _______.

12. In a sample of 832 high-school seniors, 42% admitted to having tried marijuana. A 98% confidence interval for the proportion in the population who have tried pot is the interval _____ to _____.

13. In a pilot survey we found that 57% of women with small children hold jobs; if we wished to construct a 95% confidence interval for the proportion in the population of women with small children who hold jobs, and if we wished to hold the error in the estimate to 0.5%, we would need a sample of at least \( n = \) _______.

14. The t distribution should be preferred to the z distribution in constructing confidence intervals whenever ________; in this case, if \( n < 30 \), to use a t value legitimately we must also know that ________.

15. The t distribution differs from the z distribution in that ________.
16. True or false? For a given level of confidence, a confidence interval constructed using the t distribution will be narrower than one constructed using the z distribution.

17. Find the correct t value for each of the following confidence intervals: 95% confidence, n = 23; 90% confidence, n = 12; 98% confidence, n = 9; 90% confidence, n = 55; 99% confidence, n = 89.

18. Weight gain over a month was recorded for 15 hamsters fed Lo-Pro three times a day. The result was a mean weight gain of 3 oz. with s = 0.8 oz. A 95% confidence interval for the mean weight gain of hamsters on this diet, constructed using the t distribution, will have endpoints _________ and ________.

19. The times required for a sample of five children to run 100 yd. were 18, 22, 23, 17, and 25 seconds; the population of such times is normally distributed. Using the t distribution, find a 99% confidence interval for μ, the mean time all children will require to run 100 yards.

20. We would like to give an interval estimate of the mean height of trees on Granny's Christmas tree farm. We do not know the standard deviation of the population distribution, but we strongly suspect that the distribution is normal. We have measured 18 trees and found that they have ⎯ x = 10 ft. with s = 1.5 ft. To construct a confidence interval, we should use the ________ distribution.

21. We wish to measure the average muzzle velocity of a lot of 155 mm cannon shells; we know that for such shells σ = 22 feet per second and that the distribution is normal. We have fired a sample of 14 shells and measured their muzzle velocity; in constructing our confidence interval we should use the __________ distribution.

22. We know that human weights are normally distributed, and we have a sample of 47 10-year-old boys from a large elementary school. We want to use their weights to construct a confidence interval for the mean weight of all the 10-year-old boys at the school. In constructing our interval, we should use the ________ distribution.

23. We want to make an interval estimate of the number of times a month the average student orders a pizza delivered; we suspect that the distribution is skewed upwards – that is, that a few students do it very often. We contact a sample of 47 students and from this sample calculate both x and s. In constructing our confidence interval, we should use the __________ distribution.

24. From a sample of 12 female students, we wish to make an interval estimate of the average weight loss of females on a particular exercise and diet plan. We do not know the population standard deviation or the shape of the population, and we have no reason to believe that it is normal. To construct a confidence interval, we should ____________.
Answers:
1. point, $\mu$; point, $\sigma$; point, $\sigma_x$ or standard error
2. false
3. interval estimates allow us to give assessment of reliability; and to make explicit the variability inherent in sampling
4. The population mean $\mu$ will lie within 95% of all intervals of this length centered on $\bar{x}$.
5. increases
6. increasing sample size
7. 2.33; 1.96; 0.97
8. 6.13; 84.9 to 97.1
9. 87
10. 16.8, 7.94, 2.511; $t$; 12.2 to 21.4.
11. 0.29; 0.0219; 29% $\pm$ 4.3%
12. 38% to 46%
13. 37,664
14. population standard deviation $\sigma$ is unknown; population is normally distributed
15. fatter tails; $t$ is more dispersed, and $t > z$ for any given confidence level
16. false
17. 2.074; 1.796; 2.896; 1.674; 2.633
18. 2.56 to 3.44
19. 14.02 to 27.98 sec or 21 $\pm$ 6.98
20. $t$
21. $z$
22. $t$
23. $t$
24. take a larger sample