Chapter 6: Hypothesis Testing with R - part A

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5 Step Procedure For Testing Hypotheses

The following procedures are recommended for solving test of hypothesis-type problems. The procedures allow others to follow the steps one takes in reaching a statistical decision.

1. State the null and alternative hypotheses.
2. Select an appropriate test statistic.
3. Determine the sampling distribution of the standardized test statistic under the assumption that the null hypothesis is true.
4. Calculate the $p$-value and determine if the evidence warrants rejecting the null hypothesis.
5. State in plain English what the conclusion you reached in step 4 means.
Test

Consider a school psychologist who administers the Stanford Binet IQ test to 19 randomly selected fifth graders from Watauga county and is interested in testing whether or not the IQ in the county is the same as the IQ in the general population of fifth graders. From many samples, the mean and standard deviation for the Stanford Binet have been determined to be 100 and 16 respectively. The scores for the 19 students on the Stanford Binet IQ test are: 100 101 110 106 103 128 96 96 97 106 88 104 99 102 92 97 100 109 100.
Read the data into R

> IQ <- scan()
1: 100 101 110 106 103 128 96 96 97
10: 106 88 104 99 102 92 97 100 109 100
20:
Read 19 items
> # OR
> IQ <- c(100,101,110,106,103,128,96,96,97,
+ 106,88,104,99,102,92,97,100,109,100)

For data in your book, make sure you have the BSDA package installed and loaded. To work with a particular data set from BSDA simply type attach(Data set Name) at the R prompt.

> library(BSDA) # loads BSDA
> attach(Absent) # attaches Absent
Check Normality with `qqnorm()` or `qq.plot()`

Q-Q plot was created by typing `qq.plot(IQ)` at the R prompt. Note: `qq.plot()` is a function in the `car` package. Other tools for assessing normality include `EDA()` and `ntester()` from the `BSDA` package.
Step 1:

\[ H_O : \mu = 100 \]
\[ H_A : \mu \neq 100 \]

Note that a two tailed alternative hypothesis is used since the psychologist has indicated no reason to believe Watauga county students will be either above or below the population mean IQ of 100.
Step 2:

The test statistic $\bar{X}$ is selected to test the null hypothesis.
Step 3:

Since the standard deviation of the population, \( \sigma \), and the mean under the null hypothesis are known, the sampling distribution of \( \bar{X} \) can be described as normal with mean 100 and standard deviation \( \sigma / \sqrt{n} = 16 / \sqrt{19} = 3.67 \). Recall that when sampling from a population that is normal the resulting sampling distribution of \( \bar{X} \) is also normal. The Q-Q plot simply reinforces our belief that the underlying population (IQ scores) follows a normal distribution. Note that the standardized test statistic, \( Z \), where \( Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \) follows the standard normal distribution. The value of the test statistic is

\[
Z_{\text{obs}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{101.7895 - 100}{16 / \sqrt{19}} = 0.4875084.
\]

R-cmds

```r
> z.obs <- (mean(IQ)-100)/(16/sqrt(19))
> z.obs
[1] 0.4875084
```
Step 4:

The $p$-value is calculated as
\[2 \times \mathbb{P}(Z \geq z_{\text{obs}} = 0.4875084) = 2 \times 0.3129490 = 0.6258981.\]

The large $p$-value (0.6259) indicates that observing values as inconsistent as 0.487 or more with the null hypothesis is very likely (62.59% of the time). Consequently, the statistical decision based on the $p$-value is to “fail to reject the null hypothesis.”

R-cmmands

\[
> \ p.\text{value} \leftarrow 2 * (1-\text{pnorm}(z.\text{obs})) \\
> \ p.\text{value} \\
[1] \ 0.6258981
\]
Step 5:

There is no statistical evidence to suggest the IQ scores of the fifth graders are not 100. (Note that we are not saying the IQ scores are 100. We are simply stating that the sample evidence was not sufficient to conclude the scores were not 100.)
Using the function `z.test()` in BSDA

```r
> z.test(x=IQ,sigma.x=16,mu=100)
```

One-sample z-Test

data:  IQ
z = 0.4875, p-value = 0.6259
alternative hypothesis: true mean is not equal to 100
95 percent confidence interval:  
94.59513 108.98382
sample estimates:
mean of x
  101.7895

Remember that `z.test()` has two required arguments. They are: `x=data` and `sigma.x=σ`. For additional help see the help file by typing `?z.test`. 
Testing $\mu$ when $\sigma$ Is Unknown: The $t$-Test

Consider an experiment where a medical researcher randomly selects 15 healthy college students and measures their temperatures. The researcher believes that the average temperature for college students is less than the standard 98.6 degrees Fahrenheit. The students’ temperatures were all measured at the same time of day and are given to the nearest tenth of a degree: {98.0, 98.1, 98.2, 98.2, 98.2, 98.3, 98.3, 98.3, 98.3, 98.3, 98.4, 98.4, 98.4, 98.4, 98.4, 98.6}. Can we support the researcher’s belief?
Read the data into R

```r
> TEMP <- c(98.0, 98.1, 98.2, 98.2, 98.2, 98.3, 
+ 98.3, 98.3, 98.3, 98.3, 98.4, 98.4, 98.4, 98.4, 98.6)
> TEMP
[1] 98.0 98.1 98.2 98.2 98.2 98.3 98.3 98.3 
[9] 98.3 98.3 98.4 98.4 98.4 98.4 98.6
```
Check Normality with `qqnorm()` or `qq.plot()`

Q-Q plot was created by typing `qq.plot(TEMP)` at the R prompt. Note: `qq.plot()` is a function in the `car` package. Other tools for assessing normality include `EDA()` and `ntester()` from the `BSDA` package.
Performing Exploratory Data Analysis

- Alan's Notes
- 5 Step Procedure For Testing Hypotheses
- Testing $\mu$ When $\sigma$ Is Known: The $Z$-Test
- Read the data into R
- Check Normality with `qqnorm()` or `qq.plot()`

**Step 1:**
- Using the function `z.test()` in BSDA

**Step 2:**
- Testing $\mu$ when $\sigma$ Is Unknown: The $t$-Test
- Read the data into R
- Check Normality with `qqnorm()` or `qq.plot()`

**Step 3:**
- Performing Exploratory Data Analysis

**Step 4:**
- R Commands

```
> par(mfrow=c(2,1))
> plot(density(TEMP), col="blue", lwd=2, main="")
> title(main="Density plot of TEMP")
> boxplot(TEMP, col="blue", main="", horizontal=T)
> title(main="Boxplot of TEMP")
> par(mfrow=c(1,1))
```

**Step 5:**
- That’s it for the first Part
Step 1:

\[ H_O: \mu = 98.6 \]
\[ H_A: \mu < 98.6 \]

Note that a one tailed alternative hypothesis is used since the medical researcher suspects the true mean to be less than 98.6 degrees Fahrenheit.
Step 2:

The test statistic $\bar{X}$ is selected to test the null hypothesis. Prior to selecting $\bar{X}$ as the test statistic, exploratory data analysis including appropriate Q-Q plots was used to confirm the distribution as relatively symmetric without outliers.
Step 3:

The standardized test statistic is \( t = \frac{\bar{X} - \mu}{s/\sqrt{n}} \) which follows the \( t \) distribution with \( n - 1 \) degrees of freedom. The value of the standardized test statistic is

\[
\begin{align*}
t_{\text{obs}} &= \frac{\bar{x} - \mu}{s/\sqrt{n}} \\
&= \frac{98.2933 - 98.6}{0.1438/\sqrt{15}} \\
\end{align*}
\]

R-cmds

```r
> t.obs <- (mean(TEMP)-98.6)/(sd(TEMP)/sqrt(15))
> t.obs
[1] -8.261844
```
Step 4:

The $p$-value is calculated as
$$P(t_{14} \leq t_{\text{obs}} = -8.26) = 0.000000469.$$ A small $p$-value such as 0.000000469 indicates that observing values as extreme as $-8.26$ or more is extremely unlikely when the true mean is 98.6 degrees Fahrenheit. In other words, the observed sample is extremely inconsistent with the null hypothesis. Therefore, we reject the null hypothesis $H_0 : \mu = 98.6$ in favor of the alternative hypothesis ($H_A : \mu < 98.6$).

R-commands

```r
> p.value <- pt(t.obs, 14)
> p.value
[1] 4.697174e-07
```

Note that the $p$-value reported by R $4.697174e-07$ is the same thing as $4.697174 \times 10^{-7} \approx 0.$
Step 5:

There is strong statistical evidence to suggest the average temperature of college students is less than 98.6 degrees Fahrenheit.
Using the function `t.test()`

```r
> t.test(x=TEMP, mu=98.6, alternative="less")
```

One Sample t-test

data:  TEMP
t = -8.2618, df = 14, p-value = 4.697e-07
alternative hypothesis: true mean is less than 98.6
95 percent confidence interval:
    -Inf 98.35871
sample estimates:
mean of x
  98.29333

For this type of problem, `t.test()` has one required argument `x=data`. For additional help, see the help file by typing `?t.test`. 
Thats it for the first Part

Click here for the first Part